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AN INVENTORY MODEL FOR DECAYING ITEMS HAVING STOCK DEPENDENT DEMAND UNDER THE EFFECT OF INFLATION WITH PARTIAL BACK LOGGING

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ABSTRACT

Inventory models in which the demand rates depends on the inventory level are based on the common real- life observations that greater product availability tends to stimulate more sales. This study comprises a situation in which items deteriorates and are having stock dependent demand rate. In this study, we present an optimization framework to derive an optimal ordering quantity for items with a stock dependent demand rate under inflationary conditions and partial backlogging.

INTRODUCTION

In many real life situations, especially for fashionable commodities and seasonal products, the willingness for customers to wait for back logging during a shortage period is declining with the length of the waiting time. The longer the waiting time is, the smaller the back logging rate would be. Abad (1996) considered the pricing and lot-sizing problem of perishable products for a reseller with a general decreasing partial back logging rate. A stock out occurs whenever insufficient stock exists to fulfill a replenishment order. During the stock out period, either all the demand is back ordered, in which all customers wait until their demand is satisfied; or all the demand is lost. However, in many real inventory systems, demand can be captive partially, for customers whose needs are not crucial at that time can wait for the item to be replenished, while others who can not wait will fill their demands from some other sources. The cost for a lost sale ranges from profit loss on the sale to some unspecifiable loss of good will. On the other hand, the back ordering could result in handling cost, expediting cost, and frequently special shipping cost to reduce the lead time. In order to compensate customers for the inconvenience of waiting, the idleness of equipment, or even lost production during the stock out period, the supplier may offer a variable price discount on the stock out item depending on the seriousness of the back order condition. Thus both are the backorder discount and the lead time appear to be negotiable in such a way that the supplier may cut down the present and future profit losses and the customers may be able to get the item as soon as possible to resume the production.

Chang and Dye (1999) recently developed an inventory model in which the proportion of customers who would like to accept back logging is the reciprocal of a linear function of the waiting time. Concurrently, Papachristos and S. Kouri (2000) established a partially backlogged inventory model with deterministic varying demand and constant deterioration rate, in which the backlogging rate decreases exponentially as the waiting time increases. Later, several related articles were presented, dealing with such inventory problem, such as Abad (2001), Goyal and Giri (2001), Papachristos and Skouri (2003), Teng et. al. (2002, 2003), Wang (2002).

In the present model we consider the perishable items under the effect of Inflation with partial backlogging. This model will help purchaser in deciding optimal ordering quantity under the effect of inflation and partial lost, since demand is stock dependent demand, the demand increases with the increase of stock. The environment of the whole study has been taken as inflationary, as any study done otherwise cannot justify itself under any circumstances.

ASSUMPTIONS AND LIMITATIONS

The mathematical model in this paper is developed on the basis of the following assumptions and limitations.

(1) The demand rate D(t) at time t is

$$D(t)=\begin{cases}\alpha+\beta I(t) & I(t)>0\\ \alpha & \alpha(t)\le 0\end{cases}$$

Where a and b are positive constants and I(t) is the inventory level at time t.

(2) CVT (t_1, T) is the total variable cost per unit time.

(3) We have assumed that the lead-time is zero and replenishment rate is infinite.

(4) It is assumed that only a fraction of demand is backlogged. The longer the waiting time is, the smaller is the backlogging. B(t) denote this backlogging rate, where t is the waiting time up to the next replenishment. $B(t)=\dfrac{1}{(1+\delta t)}$, Where δ is the backlogging parameter

NOTATIONS

The mathematical model in this paper is developed on the basis of the following notations.

t_d = length of time in which the product has no deterioration.

θ = Constant rate of deterioration.

t_1 = length of time (there is no shortage)

T = length of order cycle.

Q = order quantity per cycle.

A = ordering cost per order

C_1 = holding cost per unit time

C_2 = deteriorating cost per unit.

C_3 = shortage cost per backlogged items,

C_4 = unit cost of lost sales.

MATHEMATICAL MODEL

In this model we have consider replenishment model for items with constant deterioration rate and partial back logging. In this model is stock dependent demand is considered. Initially we have I_{max} units of items. During the time period [0, t_d]. The inventory level is decreasing only owing to stock-dependent demand rate. Inventory is depleted by demand as well as deterioration during the time interval (t_d , t_1). The mathematical set up of the given inventory model is given by

$$\frac{dI_1(t)}{dt}=-\left[\alpha+\beta I_1(t)\right], \qquad 0\le t\le t_d \tag{1}$$

Or $\dfrac{dI_1(t)}{dt}+\beta I_1(t)=-\alpha$

Solution with the boundary conditions t = 0, $I_{max} = I_1(t) = I_1(0)$ is given by

$$I_1(t)e^{\beta t}=-\frac{\alpha}{\beta}e^{\beta t}+I_{max}+\frac{\alpha}{\beta}$$

$$I_1(t)e^{\beta t}=\frac{\alpha}{\beta}(1-e^{\beta t})+I_{max}$$

$$I_1(t)=\frac{\alpha}{\beta}(e^{\beta t}-1)+I_{max}e^{-\beta t}, \qquad 0\le t\le t_d \tag{2}$$

Inventory is depleted by demand as well as deterioration during the time interval [t_d, t_1]. Thus, the differential equation representing the inventory status is given by

$$\frac{dI_2(t)}{dt}+\theta I_2(t)=-\left[\alpha+\beta I_2(t)\right], \qquad t_d\le t\le t_1. \tag{3}$$

$$\frac{dI_2(t)}{dt}+(\theta+\beta)I_2(t)=-\alpha$$

The solution of the given differential equation with the boundary condition when $t = t_1$, $I_2(t) = 0$ is given by

$$I_2(t) = \frac{\alpha}{\theta+\beta}\left[e^{(\theta+\beta)(t_1-t)} - 1\right] \quad (.4)$$

Continuity of I(t) at $t = t_d$, from equation (4.2) and (4.4) one can get

$$I_1(t_d) = e^{-\beta td} I_{max} - \frac{\alpha}{\beta}(1 - e^{-\beta t_d}) = \frac{\alpha}{\theta+\beta}\left[e^{(\theta+\beta)(t_1-t_d)} - 1\right]$$

Implies that maximum inventory level for each cycle is

$$I_{max} = \frac{\alpha}{(\theta+\beta)}\left[e^{(\theta+\beta)(t_1-t_d)} - 1\right]e^{\beta t_d} + \frac{\alpha}{\beta}\left(e^{\beta t_d} - 1\right) \quad (5)$$

putting the value equation (4.5) into equation (4.2), we get

$$I_1(t) = \frac{\alpha}{(\theta+\beta)}\left[e^{(\theta+\beta)(t_1-t_d)} - 1\right]e^{-\beta(t-t_d)} + \frac{\alpha}{\beta}\left[e^{-\beta(t-t_d)} - 1\right], \qquad 0 \le t \le t_d \quad .(6)$$

During the shortage, demand is partially backlogged. Thus the inventory level at time t is given by the following differential equation

$$\frac{dI_3(t)}{dt} = -\alpha\beta(T-t) = -\frac{\alpha}{1+\delta(T-t)}, t_1 \le t \le T \quad (7)$$

$$\frac{dI_3(t)}{dt} = -\frac{\alpha}{\left[1+\delta(T-t\right]}$$

$$dI_3(t) = -\frac{\alpha}{\left[1+\delta(T-t\right]}dt$$

$$I_3(t) = -\alpha\frac{\ln\left[1+\delta(T-t\right]}{-\delta} + c$$

$$I_3(t) = \frac{\alpha}{\delta}\ln\left[1+\delta(T-t\right] + c$$

with boundary conditions $t = t_1$, $I_3(t_1) = 0$

$$I_3(t) = -\frac{\alpha}{\delta}\ln\left[1+\delta(T-t_1)\right] + \frac{\alpha}{\delta}\ln\left[1+\delta(T-t)\right]$$

$$I_3(t) = -\frac{\alpha}{\delta}\left\{\ln\left[1+\delta(T-t_1)\right] - \ln\left[1+\delta(T-t)\right]\right\}, \qquad t_1 \le t \le T \quad (8)$$

Putting t=T in equation (8), the maximum amount of demand backlogged per cycle we get

$$S = -I_3(T) = \frac{\alpha}{\delta}\ln\left[1+\delta(T-t_1)\right] \quad (.9)$$

The order quantity, Q from equation (5) and (9)

$$Q = I_{max} + S$$

$$= \frac{\alpha}{\theta+\beta}\left[e^{(\theta+\beta)(t_1-t_d)} - 1\right]e^{\beta t_d} + \frac{\alpha}{\beta}(e^{\beta t_d} - 1) + \frac{\alpha}{\delta}\ln\left[1+\delta(T-t_1)\right] \quad (10)$$

The total variable cost per cycle consists of the following five elements:

(a) The ordering cost per cycle is A

(b) The inventory holding cost per cycle we get

$$Hc = c_1\left[\int_0^{t_d}(1+rt)I_1(t)dt + \int_{t_d}^{t_1}(1+rt)I_2(t)dt\right]$$

$$H_C = c_1\left[\int_0^{t_d}(1+rt)\left\{\frac{\alpha}{\theta+\beta}(e^{(\theta+\beta)(t_1-t_d)}-1)e^{-\beta(t-t_d)}+\frac{\alpha}{\beta}(e^{-\beta(t-t_d)}-1)\right\}dt\right.$$

$$\left.+\int_{t_d}^{t_1}(1+rt)\frac{\alpha}{\theta+\beta}(e^{(\theta+\beta)(t_1-t)}-1)dt\right]$$

$$=c_1\left[\int_0^{t_d}\left\{\frac{\alpha}{(\theta+\beta)}(e^{(\theta+\beta)(t_1-t_d)}-1)e^{-\beta(t-t_d)}+\frac{\alpha}{\beta}(e^{-\beta(t-t_d)}-1)\right\}dt\right.$$

$$\left.+\int_{t_d}^{t_1}(1+rt)\frac{\alpha}{\theta+\beta}(e^{(\theta+\beta)(t_1-t)}-1)dt\right]$$

$$+rc_1\left[\int_0^{t_d}\left\{\frac{t\alpha}{\theta+\beta}\left(e^{(\theta+\beta)(t_1-t_d)}-1\right)e^{-\beta(t-t_d)}+\frac{t\alpha}{\beta}(e^{-\beta(t-t_d)}-1)\right\}dt\right.$$

$$\left.+\int_0^{t_d}\frac{t\alpha}{(\theta+\beta)}(e^{(\theta+\beta)(t_1-t)}-1)dt\right]$$

$$+\frac{\alpha}{(\theta+\beta)}\left(e^{(\theta+\beta)(t_1-t_d)}-1\right)\frac{e^{\beta t_d}}{\beta^2}+\frac{t_d\alpha}{\beta}\left(-\frac{1}{\beta}-t_d\right)-\frac{\alpha}{\beta}\left(\frac{1}{\beta^2}-\frac{t_d{}^2}{2}\right)$$

$$K_1+rc_1\left[\frac{t_d\alpha}{(\theta+\beta)}(e^{(\theta+\beta)(t_1-t_d)}-1)\left(\frac{1}{-\beta}\right)-\frac{\alpha}{(\theta+\beta)}\left(\frac{e^{(\theta+\beta)(t_1-t_d)}}{\beta^2}\right)\right.$$

$$+\frac{\alpha}{\beta}\left(\frac{e^{\beta t_d}}{\beta^2}\right)+\frac{t_1\alpha}{(\theta+\beta)}\left(-\frac{1}{(\theta+\beta)}-t_1\right)-\frac{t_d\alpha}{(\theta+\beta)}\left(\frac{e^{(\theta+\beta)(t_1-t_d)}}{-(\theta+\beta)}-t_\alpha\right)$$

$$\left.-\frac{\alpha}{(\theta+\beta)}\left(\frac{1}{(\theta+\beta)^2}-\frac{t_1{}^2}{2}\right)+\frac{\alpha}{(\theta+\beta)}\left(\frac{e^{(\theta+\beta)(t_1-t_d)}}{(\theta+\beta)^2}-\frac{t_d{}^2}{2}\right)\right]$$

Where $K_1=c_1\left[\int_0^{t_d}\left\{\frac{\alpha}{\theta+\beta}\left(e^{(\theta+\beta)(t_1-t_d)}-1\right)e^{-\beta^{(t-t_d)}}+\frac{\alpha}{\beta}\left(e^{-\beta(t-t_d)}-1\right)\right\}+dt\right.$

$$\left.+\int_{t_d}^{t_1}\frac{\alpha}{(\theta+\beta)}\left(e^{(\theta+\beta)(t_1-t_d)}-1\right)dt\right]$$

$$=K_1+rc_1\left[-\frac{\alpha t_d}{\beta(\theta+\beta)}\left(e^{(\theta+\beta)(t_1-t_d)}-1\right)-\frac{\alpha}{\beta^2(\theta+\beta)}\left(e^{(\theta+\beta)(t_1-t_d)}-1\right)\right.$$

$$+\frac{\alpha}{\beta^2(\theta+\beta)}\left(e^{(\theta+\beta)(t_1-t_d)}-1\right)e^{\beta t_d}-\frac{\alpha t_d}{\beta^2}-\frac{\alpha t_d{}^2}{\beta}+\frac{\alpha e^{\beta t_d}}{\beta^3}$$

$$-\frac{\alpha}{\beta^3}+\frac{\alpha t_d{}^2}{2\beta}-\frac{\alpha t_1}{(\theta+\beta)^2}-\frac{\alpha t_1{}^2}{(\theta+\beta)}+\frac{\alpha t_d e^{(\theta+\beta)(t_1-t_d)}}{(\theta+\beta)^2}+\frac{\alpha t_d{}^2}{(\theta+\beta)}$$

$$\left.-\frac{\alpha}{(\theta+\beta)^3}+\frac{\alpha t_1{}^2}{2(\theta+\beta)}+\frac{\alpha e^{(\theta+\beta)(t_1-t_d)}}{(\theta+\beta)^3}-\frac{\alpha t_d{}^2}{2(\theta+\beta)}\right]$$

$$=K_1+rc_1\left[\frac{\alpha}{(\theta+\beta)}e^{(\theta+\beta)(t_1-t_d)}\left\{-\frac{t_d}{\beta}-\frac{1}{\beta^2}+\frac{e^{\beta t_d}}{\beta^2}+\frac{t_d}{(\theta+\beta)}+\frac{1}{(\theta+\beta)^2}\right\}\right.$$

$$+\frac{\alpha t_d}{\beta(\theta+\beta)}+\frac{\alpha}{\beta^2(\theta+\beta)}-\frac{\alpha e^{\beta t_d}}{\beta^2(\theta+\beta)}-\frac{\alpha t_d}{\beta^2}-\frac{\alpha t_d{}^2}{\beta}-\frac{\alpha}{\beta^3}+\frac{\alpha t_d{}^2}{2\beta}+\frac{\alpha e^{\beta t_d}}{\beta^3}$$

$$-\frac{\alpha t_1}{(\theta+\beta)^2}-\frac{\alpha t_1{}^2}{2(\theta+\beta)}+\frac{\alpha t_d{}^2}{(\theta+\beta)}-\frac{\alpha}{(\theta+\beta)^3}-\frac{\alpha t_d{}^2}{2(\theta+\beta)}\;\Big]$$

$$=K_1+rc_1\left[\frac{\alpha}{(\theta+\beta)}e^{(\theta+\beta)(t_1-t_d)}\left\{\frac{e^{\beta t_d}-1}{\beta^2}-\frac{\theta t_d}{\beta(\theta+\beta)}+\frac{1}{(\theta+\beta)^2}\right\}\right.$$

$$\left.-\frac{\alpha\theta t_d}{\beta^2(\theta+\beta)}+\frac{\alpha(1-e^{\beta t_d})}{\beta^2(\theta+\beta)}-\frac{\alpha\theta t_d{}^2}{2\beta(\theta+\beta)}-\frac{\alpha}{\beta^3}(1-e^{\beta t_d})-\frac{\alpha t_1}{(\theta+\beta)^2}-\frac{\alpha t_1^2}{2(\theta+\beta)}-\frac{\alpha}{(\theta+\beta)^3}\right]$$

$$Hc=K_1+rc_1\left[\frac{\alpha}{(\theta+\beta)}e^{(\theta+\beta)(t_1-t_d)}\left\{\frac{e^{\beta t_d}-1}{\beta^2}-\frac{\theta t_d}{\beta(\theta+\beta)}+\frac{1}{(\theta+\beta)^2}\right\}\right.$$

$$\left.-\frac{\alpha\theta t_d}{\beta^2(\theta+\beta)}-\frac{\alpha\theta(1-e^{\beta t_d})}{\beta^3(\theta+\beta)}-\frac{\alpha\theta t_d{}^2}{2\beta(\theta+\beta)}-\frac{\alpha t_1}{(\theta+\beta)^2}-\frac{\alpha t_1{}^2}{2(\theta+\beta)}-\frac{\alpha}{(\theta+\beta)^3}\right] \quad (11)$$

The deterioration cost per cycle we get

$$Dc=C_2\left[I_2(t_d)(1+rt)-\int_{t_d}^{t_1}D(t)(1+rt)dt\right]$$

$$=C_2\left\{\frac{\alpha(1+rt}{(\theta+\beta)}\left[e^{(\theta+\beta)(t_1-t_d)}-1\right]-\int_{t_d}^{t_1}\alpha+\frac{\beta\alpha}{(\theta+\beta)}\left[e^{(\theta+\beta)(t_1-t)}-1\right](1+rt)dt\right\}$$

$$=C_2\left\{\frac{\alpha(1+rt)}{(\theta+\beta)}\left[e^{(\theta+\beta)(t_1-t_d)}-1\right]-\int_{t_d}^{t_1}\alpha+\frac{\alpha\beta}{(\theta+\beta)}\left[e^{(\theta+\beta)(t_1-t)}-1\right]dt\right.$$

$$\left.-r\int_{t_d}^{t_1}\alpha+\frac{\alpha\beta}{(\theta+\beta)}\left[e^{(\theta+\beta)(t_1-t)}-1\right]dt\right\}$$

$$=C_2\left\{\frac{\alpha(1+rt)}{(\theta+\beta)}\left[e^{(\theta+\beta)(t_1-t_d)}-1\right]-\alpha t_1+\frac{\alpha\beta}{(\theta+\beta)^2}+\frac{\alpha\beta t_1}{(\theta+\beta)}+\alpha t_d\right.$$

$$-\frac{\alpha\beta e^{(\theta+\beta)(t_1-t_d)}}{(\theta+\beta)^2}-\frac{\alpha\beta t_d}{(\theta+\beta)}-r\left[\left(\frac{\alpha}{2}-\frac{\alpha\beta}{(\theta+\beta)}+\frac{1}{2}\right)t_1{}^2-(1+\alpha)\frac{t_d{}^2}{2}\right.$$

$$\left.\left.-\frac{\alpha\beta t_1}{(\theta+\beta)^2}+\frac{\alpha\beta e^{(\theta+\beta)(t_1-t_d)}}{(\theta+\beta)^2}-\frac{\alpha\beta}{(\theta+\beta)^3}+\frac{\alpha\beta e^{(\theta+\beta)(t_1-t_d)}}{(\theta+\beta)^3}+\frac{\alpha\beta t_d}{(\theta+\beta)}\right]\right\} \quad (12)$$

The shortage cost per cycle due to partial backlogging is given by

$$Sc=c_3\int_{t_1}^{T}(1+rt)(-I_3(t))dt$$

$$=c_3\int_{t_1}^{T}(1+rt)\frac{\alpha}{\delta}\left\{\ln\left[1+\delta\left(T-t_1\right)\right]-\ln\left[1+\delta\left(T-t\right)\right]\right\}dt$$

$$=\frac{\alpha c_3}{\delta}\left[\int_{t_1}^{T}\left\{\ln\left[1+\delta\left(T+t_1\right)\right]-\ln\left[1+\delta\left(T-t\right)\right]\right\}\right]dt+\frac{c_3\alpha}{\delta}r\left[\int_{t_1}^{T}t\left\{\ln\left[1+\delta\left(T-t_1\right)\right]\right.\right.$$

$$-\ln\left[1+\delta\left(T-t\right)\right]\Big\}dt\Big]$$

$$=K_2+\frac{c_3\alpha r}{\delta}\left[\left(\frac{T^2-t_1^2}{2}\right)\ln\left[1+\delta\left(T-t_1\right)\right]+\frac{t_1^2}{2}\ln\left[1+\delta\left(T-t_1\right)\right]\right.$$

$$\left.-\frac{\delta}{2}\int_{t_1}^{T}\frac{t^2}{\left[1+\delta\left(T-t\right)\right]}dt\right]$$

Where $K_2=\frac{c_3\alpha}{\delta}\left\{\left(T-t_1\right)-\frac{\ln\left[1+\delta\left(T-t_1\right)\right]}{\delta}\right\}$

$$=K_2+\frac{c_3\alpha r}{\delta}\left[\frac{(T^2-t_1^2)}{2}\ln\left[1+\delta\left(T-t_1\right)\right]+\frac{t_1^2}{2}\ln\left[1+\delta\left(T-t_1\right)\right]\right.$$

$$\left.+\frac{\delta}{2}\int_{t_1}^{T}\left\{\frac{t}{\delta}+\frac{1}{\delta^2}+\frac{T}{\delta}-\frac{1}{\delta^2\left(1+\delta\left(T-t\right)\right)}-\frac{2T}{\delta\left[1+\delta\left(T-t\right)\right]}-\frac{T^2}{\left[1+\delta\left(T-t\right)\right]}\right\}dt\right]$$

$$K_2+\frac{C_3\alpha r}{\delta}\left[\frac{(T^2-t_1^2)}{2}\ln[1+\delta(T-t_1)]+\frac{t_1^2}{2}\ln[1+\delta(T-t_1)]+\frac{\delta}{2}\left\{\left(\frac{t^2}{2\delta}+\right.\right.\right.$$

$$\left.\left.\left.\frac{t}{\delta^2}+\frac{Tt}{\delta}-\frac{1}{\delta^2}\frac{\ln\left[1+\delta\left(T-t\right)\right]}{(-\delta)}-\frac{2T}{\delta}\frac{\ln\left[1+\delta\left(T-t\right)\right]}{(-\delta)}-T^2\frac{\ln\left[1+\delta\left(T-t\right)\right]}{(-\delta)}\right)_{t_1}^{T}\right\}\right]$$

$$K_2+\frac{C_3\alpha r}{\delta}\left[\frac{(T^2-t_1^2)}{2}\ln[1+\delta(T-t_1)]+\frac{t_1^2}{2}\ln[1+\delta(T-t_1)]+\frac{\delta}{2}\left\{\frac{t^2}{2\delta}+\frac{T}{\delta^2}\right.\right.$$

$$+\frac{T^2}{\delta}-\frac{t_1^2}{2\delta}+\frac{T_1}{\delta^2}-\frac{Tt_1}{\delta}-\frac{\ln\left[1+\delta\left(T-t_1\right)\right]}{\delta^3}-\frac{2T\ln\left[1+\delta\left(T-t_1\right)\right]}{\delta^2}$$

$$\left.\left.-\frac{T^2\ln\left[1+\delta\left(T-t_1\right)\right]}{\delta}\right\}\right]$$

$$S_C=K_2+\frac{C_3\alpha r}{\delta}\left[\frac{3T^2}{4}+\frac{T}{2\delta}-\frac{T_1^2}{4}-\frac{t_1}{2\delta}-\frac{Tt_1}{2}-\left(\frac{1+2\delta T}{2\delta^2}\right)\ln[1+\delta(T-t_1)]\right]\text{.....(13)}$$

The opportunity cost per cycle due to lost sales is given by

$$OC=C_4\int_{t_1}^{T}\alpha\left[1-\beta\left(T-t\right)\right]\left(1+rt\right)dt$$

$$=C_4\int_{t_1}^{T}\alpha\left[1-\frac{1}{1+\delta\left(T-t\right)}\right]\left(1+rt\right)dt$$

$$=C_4\alpha\left[T-t_1-\frac{\ln\left[1+\delta\left(T-t_1\right)\right]}{\delta}+r\int_{t_1}^{T}t\left\{1-\frac{1}{\left[1+\delta\left(T-t\right)\right]}\right\}dt\right]$$

$$=C_4\alpha\left\{T-t_1-\frac{\ln\left[1+\delta\left(T-t_1\right)\right]}{\delta}+\frac{r\left[T^2-t_1^2\right]}{2}-r\left[\int_{t_1}^{T}-\frac{1}{\delta}+\frac{1}{\delta\left[1+\delta\left(T-t\right)\right]}\right.\right.$$

$$+\frac{T}{\left[1+\delta(T-t)\right]}\Bigg]dt\Bigg\}$$

$$=\alpha C_4\left\{T-t_1\frac{\ln\left[1+\delta(T-t_1)\right]}{\delta}+\frac{r\left(T^2-t_1^2\right)}{2}-r\left[\frac{t_1-T}{\delta}+\left(\frac{1+\delta}{\delta^2}\right)\right.\right.$$

$$\left.\left.\ln\left[1+\delta\left(T-t_1\right)\right]\right]\right\}$$

$$OC=\alpha C_4\left\{T-t_1-\frac{\ln\left[1+\delta(T-t_1)\right]}{\delta}+\frac{r\left(T^2-t_1^2\right)}{2}-r\left[\frac{t_1-T}{\delta}+\left(\frac{1+\delta}{\delta^2}\right)\right.\right.$$

$$\left.\left.\ln\left[1+\delta\left(T-t_1\right)\right]\right]\right\} \quad .(.14)$$

Therefore the total variable cost per unit time is given by.

CVT (t_1, t) = {Ordering cost+ inventory holding cost + the deterioration cost + Shortage cost +opportunity cost}/ T

$$=\left(A+HC+DC+SC+OC\right)/T.$$

$$CVT(t_1,T)=\frac{1}{T}\left[A+K_1+rc_1\left[\frac{\alpha}{(\theta+\beta)}e^{(\theta+\beta)(t_1-t_d)}\left\{\frac{e^{\beta t_d}-1}{\beta^2}-\frac{\theta t_d}{\beta(\theta+\beta)}\right.\right.\right.$$

$$\left.+\frac{1}{(\theta+\beta^2)}\right\}-\frac{\alpha\theta t_d}{\beta^2(\theta+\beta)}-\frac{\alpha\theta\left(1-e^{\beta t_d}\right)}{\beta^3(\theta+\beta)}-\frac{\alpha\theta t_d{}^2}{2\beta(\theta+\beta)}-\frac{\alpha t_1}{(\theta+\beta)^2}$$

$$\left.-\frac{\alpha t_1^2}{2(\theta+\beta)}-\frac{\alpha\theta t_d}{\beta^2(\theta+\beta)^3}\right]+C_2\left\{\frac{\alpha(1+rt)}{(\theta+\beta)}\left[e^{(\theta+\beta)(t_1-t_d)}-1\right]-\alpha t_1\right.$$

$$+\frac{\alpha\beta}{(\theta+\beta)^2}+\frac{\alpha}{(\theta+\beta)}+\alpha t_d-\frac{\alpha\beta e^{(\theta+\beta)(t_1-t_d)}}{(\theta+\beta)^2}-\frac{\alpha\beta t_d}{(\theta+\beta)}$$

$$-r\left[\left(\frac{\alpha}{2}-\frac{\alpha\beta}{(\theta+\beta)}+\frac{1}{2}\right)t_1^2-(1+\alpha)\frac{t_d{}^2}{2}-\frac{\alpha\beta t_1}{(\theta+\beta)^2}+\frac{\alpha\beta e^{(\theta+\beta)(t_1-t_d)}}{(\theta+\beta)^2}\right.$$

$$\left.\left.-\frac{\alpha\beta}{(\theta+\beta)^3}+\frac{\alpha\beta e^{(\theta+\beta)(t_1-t_d)}}{(\theta+\beta)^3}+\frac{\alpha\beta t_d}{(\theta+\beta)}\right]\right\}+K_2+\frac{C_3\alpha r}{\delta}\left\{\frac{3T^2}{4}\right.$$

$$\left.+\frac{T}{2\delta}-\frac{T_1^2}{4}-\frac{T_1}{2\delta}+\frac{Tt_1}{2}-\left(\frac{1+2\delta T}{2\delta^2}\right)\ln\left[1+\delta\left(T-t_1\right)\right]\right\}+\alpha C_4$$

$$\left\{T-t_1-\frac{\ln[1+\delta(T-t_1)]}{\delta}+\frac{r\left(T^2-t_1^2\right)}{2}-r\left[\frac{t_1-T}{\delta}+\left(\frac{1+\delta}{\delta^2}\right)\ln[1+\delta(T-t_1)]\right]\right\}$$

(15)

In this way we obtain total cost equation of the system in consideration. It is observed that the obtained total cost equation is highly non-linear transcendental equation. It can be solved numerically with the help of some suitable computational software like MATLAB or MATHEMATICA. After that we can obtain optimal values of time and thereafter-total cost of the system under discussion.

CONCLUSIONS

In this paper we have considered an inventory model for the items having stock dependent demand. It was very obvious a fact that given some time, every item can create a niche for itself in the customer's mind, hence increasing its demand with the passage of time. Later with the advent of supermarkets, it was commonly acknowledged that vast displays of stocks induce the customer into buying more. Also it was noted that a decline in the level of displayed stock witnessed a decline in the customer's demand for that item. For a long time, stock dependent demands were not explored, although the dependence of the sale of any item on its selling price is not a new concept, but a common sense conclusion. It is a general observation that an increase in the selling price of the commodity will deter its customer's from opting that item in future. However, a dip in the selling price, in whatever form it may come, always notices a sudden In this paper we have also considered the factor of inflation with constant rate of inflation. This paper helps to determine an optimal replenishment policy for non-instantaneous deteriorating items with stock dependent demand and partial backlogging.

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